ANALYSIS SYMPOSIUM

ALLEN FORTE

Introduction

This atonal work, composed some sixty years ago, is representative of a music that remains difficult to understand. Moreover, this particular piece presents special problems to the analyst. Not only is it in the "athematic" and "pointillistic" style more characteristic of the 12-tone works of Webern, for example, the Symphony, Op. 21, but it is also most unusual in that it comprises a succession of single attacks, with but one instance of a simultaneous attack in two instruments, viz., the guitar and timpani in m. 14. As a result, the basic analytical problem of segmentation – the determination of structural units – is considerably aggravated. Once the key to this is discovered, however, the difficulties disappear to a large extent.

The work is one of five posthumous orchestral pieces edited by Friedrich Cerha and published under the general editorship of Hans Moldenhauer, who in 1965 discovered the manuscripts to these and other previously unknown works by Webern. This piece resembles the others in the set hardly at all, nor does it resemble any of the pieces in Webern's Five Pieces for Orchestra, Op. 10, which, according to Moldenhauer, were composed at about the same time (1913). Specifically, the Op. 10 pieces are either denser in texture or more complicated with respect to rhythm, register, orchestration, and so on, than is the work under consideration. Op. 10/1, for example, although far denser than our subject work, presents a far more conventional surface, with a clear differentiation of primary melodic materials (including a straightforward statement of one of the hexachords prominent in the work under consideration) and secondary, accompanimental formations.

Form

Here as in all of Webern's atonal works it is necessary to distinguish between "external" and "internal" form. By external form is meant the succession of distinct parts determined by some obvious features of the music, such as change of tempo. Internal form designates the more complex configurations determined by structural pitch-class sets (pc sets) *1 and the interaction of those sets.

As determined by simultaneous rests in all instruments, the external form of the work under consideration consists of a succession of five parts: mm.1-3; mm.4-5; mm.6-8; mm. 9-12; mm.13-16. In addition, these five parts coalesce to form two larger parts determined most markedly by a radical change in register at m.10: mm.1-8 and mm.9-16.

Segmentation

Before any significant higher level observations can be made about this work it is necessary to delimit the structural components (pc sets), the segmentation procedure mentioned above. A segment generally consists of a configuration each element of which is contiguous to at least one other element of the configuration. Herein lies the problem in this particular work, however, for there are a great many such candidate segments. It is necessary to ascertain which of those segments carries a significant pc set, as distinct from segments that form sets incidental to the main structural components of the music. No attempt will be made here to formalize the segmentation procedure, but it is hoped that the reader will find the results convincing in terms of three general criteria: recurrence, consistency, and parsimony. *2 To simplify, the analysis will begin with a discussion of pc sets of size 6, and continue to sets of sizes 5 and 7 and sizes 4 and 8, taking into account, of course, not only the set occurrences but the relations among those. A synthesis of these will then be presented mainly in terms of set-complex relations, and other, supportive, aspects such as mode of performance, will be discussed later.

The Hexachords

Before examining the hexachordal structure of the piece, it is important to point out that a 12-note aggregate is unfolded at the very outset, extending (without duplication of pc) through the D played by tuba in m.5. If the latter note is regarded as the beginning of another aggregate, then a second aggregate is completed with the entrance of A played by bass clarinet in m. 7. The segmentation of these 12-note aggregates will be mentioned below in connection with the discussion of pc sets of 8 elements.

The main structural hexachords in the piece are 6-Z6 and 6-Z38. *3 Example 1 displays the ten occurrences of these sets throughout the piece. The general relations between these two pc sets will be familiar to many readers. *4 First, the two hexachords have the same total interval content distribution, as represented by the numerical array (vector) 421242, where the number of intervals of class 1 is 4, the number of intervals of class 2 is 2, . . . , and the number of intervals of class 6 is 2. Moreover, the two sets are complement-related. That is, 6-Z38 is some form of the complement of 6-Z6 and vice versa. As a specific example, consider the two sets designated A and B in Example 1. The literal complement of 6-Z6 can be represented in conventional interger notation as 7, 8, 9, 10, 2, 3 (cf. C in Example 1).

Adding 9 to each of those integers (transposing up a "major sixth") yields pc set B: 4, 5, 6, 7, 11, 0. Here Webern has deployed these two "Z-related" sets in a way that is most unusual for him (but which would not be unusual in a Schoenberg work). The sets are interlocked over the first part of the piece, forming the three parts AB, CD, AB, with E entering as the last hexachord of the first part of the piece. In a second part of the piece (m. 9ff.) there is again an interlocking pair 6-Z6/ 6-Z38 (F and G), with two forms of 6-Z38 (G and H) overlapping, as shown in Example 1.

As a result of this interlocking, some notes are common to both sets of a pair, forming locally invariant (intersection) pc sets. Most remarkable is the fact that the transpositional derivation of D from C is the same as that of B from A, with the consequence that the same type of set is held invariant between the two pairs: pc set 5-7. *5 (Other instances of 5-7 and its complement will be discussed below.) Example 2 summarizes the relevant information.

These invariant subsets effect a structural differentiation of pitches that is not evident from a cursory examination of the surface of the music. Not only is pc set 5-7 held fixed between interlocking forms of the Z-pair, but there is also a highly significant intersection involving single notes. In particular, the overlapping of B and C (6-Z38) yields one invariant pc, pc7 (G in m.4), while the overlapping of D and A (6-Z6) yields the single invariant pc, pc1 (C♯ in m.5). These two pcs, 7 and 1, are precisely the pcs that are missing from the union of the two forms of 5-7 that are held invariant between the Z-pairs, and their strategic positions in the music are clearly evident. Note especially that pc7 stands at the beginning of the second part in the external form of the music.

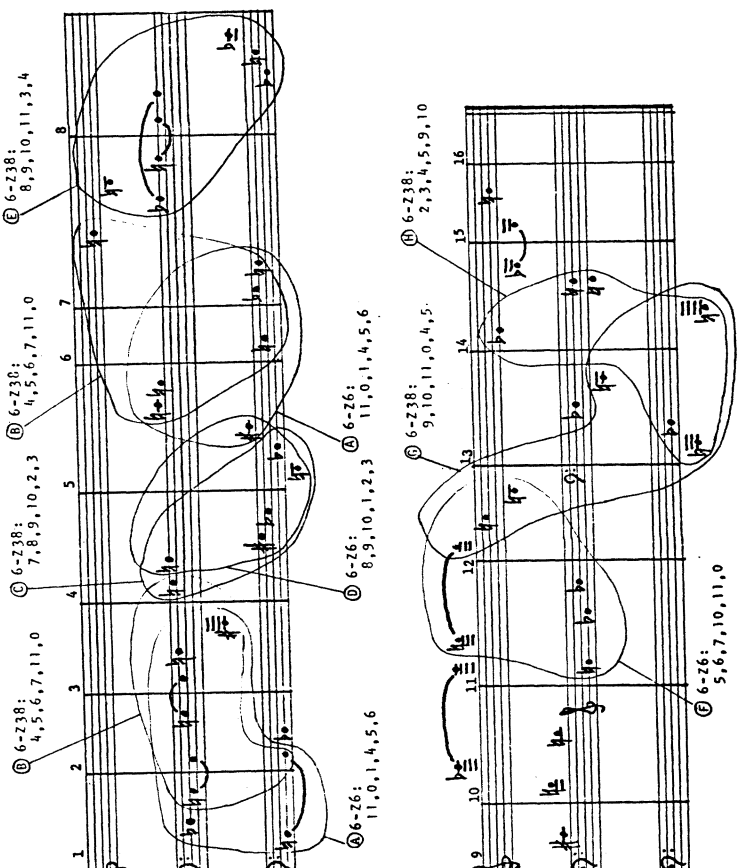
Relations between successive forms of 6-Z38 and between successive forms of 6-Z6 are shown in Example 3. From this example it can be seen that the conditions of invariance fluctuate markedly and with a certain regularity. *6 While each succession begins with minimal invariance, it ends with maximal invariance. One effect of this is to create a link between the two large sections of the work by means of a common subset of the structural pc set 6-Z6. This subset, pc set 4-9, although not formed as a contiguous segment in A, is very clearly stated as a "melodic" set in F, namely, as the violin line in the uppermost register beginning in m.10. This melodic line is one of only two such in the piece containing more than 3 notes. (The other is the hexachord played by the celesta, m.9, which will be discussed below.)

Invariants among non-consecutive forms of 6-Z38 and 6-Z6 can be read from Example 3 and will not be recited in extenso. For example, pc set 4-9 is common to the E and H forms of 6-Z38. In E this set occurs as the attack succession A, B♭, E, E♭, while in H it occurs as the subcession A, B♭, E♭, E.

Order Relations Among the Hexachords

The latter observation leads naturally to a more thorough consideration of order relations among the hexachords. Example

EXAMPLE 1


B 6-Z38: 4,5,6,7,11,0
A 6-Z6: 11,0,1,4,5,6
C 6-Z38: 7,8,9,10,2,3
D 6-Z6: 8,9,10,1,2,3
B 6-Z38: 4,5,6,7,11,0
A 6-Z6: 11,0,1,4,5,6
E 6-Z38: 8,9,10,11,3,4
G 6-Z38: 9,10,11,0,4,5
H 6-Z38: 2,3,4,5,9,10
F 6-Z6: 5,6,7,10,11,0

EXAMPLE 2

Relations Between 6-Z6 and 6-Z38

			Invariant Subset
6-Z6	A: 11, 0, 1, 4, 5, 6		
6-Z38	B: 4, 5, 6, 7, 11, 0	= $T(\overline{A}, 9)$	5-7: 11, 0, 4, 5, 6
6-Z38	C: 7, 8, 9, 10, 2, 3	= T(B, 3)	pc7
6-Z6	D: 8, 9, 10, 1, 2, 3	= $T(\overline{C}, 9)$	5-7: 8, 9, 10, 2, 3
6-Z6	A: 11, 0, 1, 4, 5, 6	= T(D, 3)	pc1
6-Z38	B: 4, 5, 6, 7, 11, 0	= $T(\overline{A}, 9)$	5-7: 11, 0, 4, 5, 6
6-Z38	E: 8, 9, 10, 11, 3, 4	= T(B, 4)	pcs 4, 11 (both A and B)
6-Z6	F: 5, 6, 7, 10, 11, 0	= $T(\overline{E}, 5)$	pcs 10, 11
6-Z38	G: 9, 10, 11, 0, 4, 5	= $T(\overline{F}, 4)$	4-6: 10, 11, 0, 5
6-Z38	H: 2, 3, 4, 5, 9, 10	= T(G, 5)	4-8: 4, 5, 9, 10

EXAMPLE 3

Relations between successive forms
of the hexachords 6-Z38 and 6-Z6

6-Z38		Invariants	Invariance Condition
B: 4,5,6,7,11,0			
C: 7,8,9,10,2,3	= T(B,3)	pc7	minimal
B: 4,5,6,7,11,0	= T(C,9)	pc7	minimal
E: 8,9,10,11,3,4	= T(B,4)	pcs 4,11	near minimal
G: 9,10,11,0,4,5	= T(E,1)	4-6: 9,10,11,4	maximal
H: 2,3,4,5,9,10	= T(G,5)	4-8: 4,5,9,10	maximal
6-Z6			
A: 11,0,1,4,5,6			
D: 8,9,10,1,2,3	= T(A,9)	pc1	minimal
A: 11,0,1,4,5,6	= T(D,3)	pc1	minimal
F: 5,6,7,10,11,0	= T(A,6)	4-9: 5,6,11,0	maximal

4 shows the ordered set in integer notation, the number of order inversions (after Babbitt) between successive forms, the intervals formed by successive notes of the set, the basic interval pattern (bip)*7, and the number of permutations of the set that will yield the particular bip. In the case of H, there is the problem of the simultaneous occurrence of F and D in m. 14 – a unique event in this piece, as remarked above. Accordingly, two bips are shown for that set, depending upon which of the two simultaneous notes is taken as "first."

Although the order relations among the hexachords do not give evidence of highly systematic treatment, there are a number of significant correspondences, indicating a certain regularity and compositional concern for order. Notice, first, that each bip contains at least two 1's. This is by means trivial, for both sets have associated with them bips that contain no 1's whatever – e.g., 45555. All but three of the bips contain at least one 2, and occurrences of intervals of classes 1 and 2 are, of course, very evident at the surface level of the music throughout. The considerable degree of similarity among bips is reflected by similarities among portions of the interval successions. Indeed, the succession 2-1-4 occurs twice, while the succession 1-5-1 occurs four times.

Perhaps of greatest interest are the subsegments in which order is preserved or nearly preserved between disjunct sets. The succession 9 10 3 4 in E becomes the succession 9 10 4 3 in H, and this is precisely the invariant tetrachord 4-9 whose importance as a linking set has been commented upon above. Between B and G (and A) the dyad 0 11 remains intact, and it is this dyad that is singled out for special attention in B, when it is played by the harp in m. 5.

Between successive forms of the same type of set there are, in most cases, a large number of order inversions, but this number is not, in general, reflected in a substantial difference with respect to bip. For example, between E and G there are seven order inversions (almost half the maximum), whereas the bips differ only with respect to the interval classes 3 and 5.

It can be concluded from the foregoing discussion that although order relations are not insignificant, they are far less important as structural means than are other aspects, notably the invariant subsets.

EXAMPLE

4

Order Relations for the Hexachords

	Ordered Set	Number of Order Inversions	Interval Succession	Bip	Number of Permutations
6-Z6					
	A: 11 1 0 4 5 6		2-1-4-1-1	11124	8
	D: 9 8 3 2 10 1	7	1-5-1-4-3	11345	8
	A: 1 0 11 4 6 5	6	1-1-5-2-1	11125	12
	F: 7 5 6 10 0 11	1	2-1-4-2-1	11224	8
6-Z38					
	B: 0 11 4 5 6 7		1-5-1-1-1	11115	4
	C: 7 9 8 3 2 10	7	2-1-5-1-4	11245	16
	B: 0 11 4 6 5 7	6	1-5-2-1-2	11225	12
	E: 8 11 9 10 4 3	11	3-2-1-6-1	11236	4
	G: 5 0 11 9 10 4	7	5-1-2-1-6	11256	16
	H: 9 10 3 4 2 5	10	1-5-1-2-3	11235	8
	9 10 3 4 5 2	9	1-5-1-1-3	11135	12

The Hexachord 6-Z44

Celesta in mm. 9-11 sets out a hexachord that remains anomalous in this piece: 6-Z44. There are no other occurrences of this set as a contiguous segment, nor are there any occurrences of its complement, 6-Z19. As has been pointed out elsewhere (see Reference 2), this set is associated with Schoenberg's name. Both Berg and Webern used it elsewhere (Berg most obviously in the Chamber Concerto, but also in the Four Pieces for Clarinet and Piano), undoubtedly as a kind of private musical tribute to their teacher. That may be its raison d'être here. Its structural relation to the other sets used in the composition will be evident in the discussion of set-complex relations below.

The Five- and Seven-Note Sets

From the segmentation shown in Example 1 it is evident that the hexachords do not account for all the notes in the piece — specifically, the B♭ in m. 2, four notes in mm. 9-10, and the final two notes. The segmentation into sets of 5 and 7 elements shown in Example 5 places these notes in the context of structural sets and indicates the ways in which the musical texture is shaped by sets of 6-1 and 6+1 elements. In fact, these sets are of only two types: 5-6 and 5-7, together with their complements, 7-6 and 7-7, respectively.

Of the two 5-note sets, 5-7 is clearly the most important. Three of the occurrences of that set shown in Example 5 have already been discussed in connection with the invariant subsets resulting from the intersection of forms of 6-Z6 and 6-Z38. The intersection of successive forms of 5-7 also yields significant sets, the most interesting of which are the 4-note sets whose complements are also represented in the music. Example 6 summarizes and provides a reference for the following discussion, together with Example 5.

The two sets 5-6 and 5-7 interact locally only in the first section of the piece. Since the common subsets are non-trivial contiguous formations and are not indicated in Example 6, it may be of interest to give them here. Common to B and D is the set 4-7: 0,4,5,6. (Other occurrences of that set and its complement, 8-7, are shown in Example 10.) Common to B and C is the trichord 3-4: 0,4,5. The remaining combinations (A & C and A & D) produce the same common set, viz., 4-8: 11,0,4,5. The sum of all these common sets is, perhaps obviously, D: 11,0,4,5,6.

EXAMPLE

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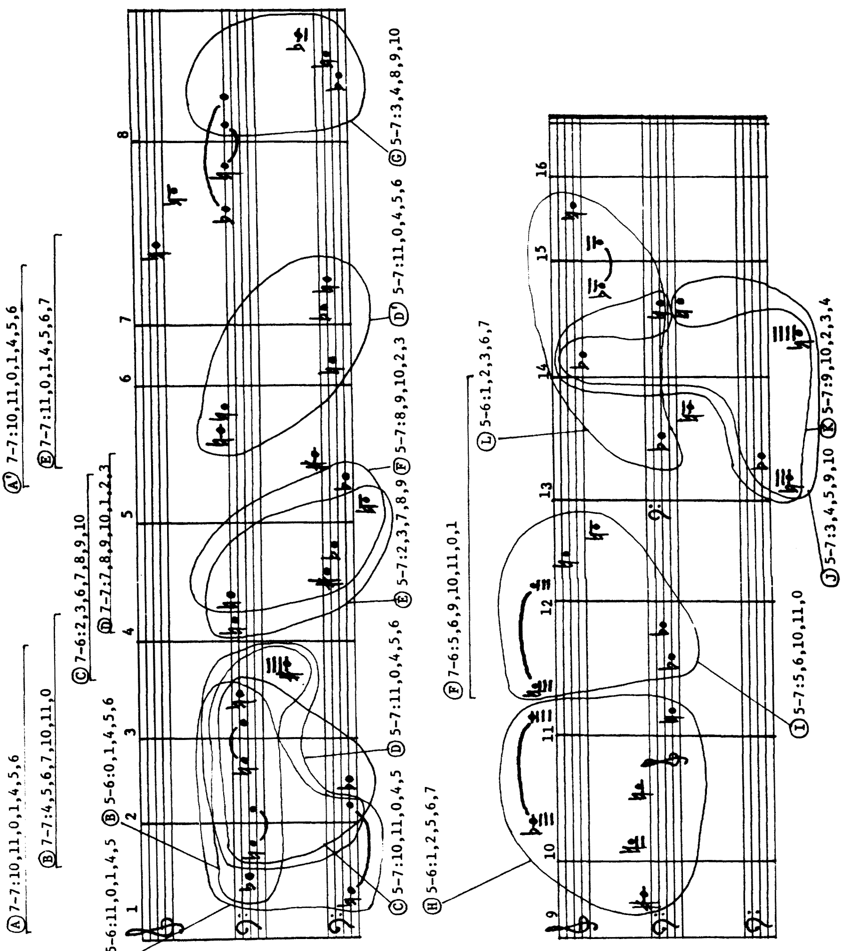

(A) 7-7:10,11,0,1,4,5,6
(B) 7-7:4,5,6,7,10,11,0
(A') 7-7:10,11,0,1,4,5,6
(E) 7-7:11,0,1,4,5,6,7
(C) 7-6:2,3,6,7,8,9,10
(D) 7-7:7,8,9,10,1,2,3
(A) 5-6:11,0,1,4,5
(B) 5-6:0,1,4,5,6
(C) 5-7:10,11,0,4,5
(D) 5-7:11,0,4,5,6
(E) 5-7:2,3,7,8,9
(F) 5-7:8,9,10,2,3
(D') 5-7:11,0,4,5,6
(G) 5-7:3,4,8,9,10
(H) 5-6:1,2,5,6,7
(F) 7-6:5,6,9,10,11,0,1
(L) 5-6:1,2,3,6,7
(I) 5-7:5,6,10,11,0
(J) 5-7:3,4,5,9,10
(K) 5-7:9,10,2,3,4

EXAMPLE 6

Relations among 5 and 7 element sets

			Invariants
5-6			
A:	11,0,1,4,5		
B:	0,1,4,5,6	= T(1(A),5)	4-7: 0,1,4,5
H:	1,2,5,6,7	= T(B,1)	3-4: 1,5,6
L:	1,2,3,6,7	= T(1(H),7)	4-8: 1,2,6,7
5-7			
C:	10,11,0,4,5		
D:	11,0,4,5,6	= T(1(C),4)	4-8: 11,0,4,5
E:	2,3,7,8,9	= T(D,3)	none
F:	8,9,10,2,3	= T(1(E),5)	4-9: 8,9,2,3
D':	11,0,4,5,6	= T(1(F),2)	none
G:	3,4,8,9,10	= T(D',4)	pc4
I:	5,6,10,11,0	= T(G,2)	pc10
J:	3,4,5,9,10	= T(1(1),3)	pcs 5,10
K:	9,10,2,3,4	= T(1(J),7)	4-9: 3,4,9,10
7-6			
C:	2,3,6,7,8,9,10		
F:	5,6,9,10,11.0.1	= T(C,3)	pcs 9,10
7-7			
A:	10,11,0,1,4,5,6		
B:	4,5,6,7,10,11,0	= T(A,6)	6-7: 4,5,6,10,11,0
D:	7,8,9,10,1,2,3	= T(B,3)	pcs 7,10
A':	10,11,0,1,4,5,6	= T(D,3)	pcs 1,10
E:	11,0,1,4,5,6,7	= T(1(A'),5)	6-Z6: 11,0,1,4,5,6

Before discussing the invariant subsets of the forms of 5-7, it may be of interest to consider complete variance — i.e., the situation in which a form of 5-7 is mapped into its literal complement, thus leaving no elements in common between two set-forms. For the general case, there are three levels of transposition that produce complete variance under the operation of inversion followed by transposition, the particular levels of transposition being determined by the specific pcs in the set that undergoes transformation. And there are two levels, not dependent upon pc content, at which the set is mapped into its complement under the operation of transposition alone, 3 and its modulo 12 inverse, 9. Any other transformation will produce at least one invariant pc, and thus a transformation that yields complete variance is very special, indeed.

In the piece under consideration, complete variance occurs among four pairs of forms of pc set 5-7, as follows:

EC	E = T(1(C), 7)
ED	E = T(D, 3)
D'F	D' = T(1(F), 2)
IE	I = T(E, 3)

Thus, three out of five possible transformations that produce complete variance are utilized. The musical effect of this, of course, is maximum differentiation or "contrast" with respect to pc content of fundamentally equivalent sets. (Since the union of each pair produces a 10-note set, it is of interest to note that one of the two excluded pcs is, in each case, pc 1.) Thus, this set of sets is markedly different with respect to the internal form of the work than is the set of sets involving invariant subsets. It can be pointed out here that the other 5-note set, 5-6, although capable of complete variance at one level of transposition preceded by inversion (and at no level under transposition alone), is not utilized in this way. That is, every pair of sets of type 5-6 shares at least one pc.

Example 7 presents an exhaustive list of the invariants among forms of 5-7, excluding those cases in which the invariant subset is empty. This shows the transformation by which the second set is derived from the first, the resulting invariants, and gives brief comments concerning the mode of occurrence of the invariants. (Contiguous means that the invariant subset is a contiguous segment in the music.) Although there are a number of non-contiguous 4-note invariant subsets, these are still of analytical significance since they are, without exception, replicated by sets of the same type at the level of sets of size 4 — which will be discussed shortly.

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EXAMPLE

7

Invariant Subsets of Forms of 5-7

Sets	Transformation	Invariants	Comment
C & D	IT4	4-8: 11,0,4,5	contiguous in C
C & F	T10	pc10	
C & G	IT8	pcs 4,10	contiguous in both same register
C & I	IT10	4-6: 10,11,0,5	contiguous in I
C & J	T5	3-5: 4,5,10	contiguous in C
C & K	IT2	pcs 4,10	contiguous in C
D & G	T4	pc4	
D & I	T6	4-9: 11,0,5,6	not contiguous
D & J	IT9	pcs 4,5	contiguous in both
D & K	T10	pc4	
E & F	IT5	4-9: 8,9,2,3	contiguous in both (identical)
E & G	T1	3-5: 3,8,9	contiguous in both pcs 8,9 same register
E & J	IT0	pcs 3,9	not contiguous
E & K	IT7	3-5: 9,2,3	not contiguous
F & G	IT6	4-6: 8,9,10,3	not contiguous

Sets	Transformation	Invariants	Comment
F & I	IT8	pc10	
F & J	T7	3-5: 9,10,3	not contiguous
F & K	IT0	4-8: 9,10,2,3	not contiguous
G & I	T2	pc10	
G & J	IT1	4-9: 3,4,9,10	contiguous in both pc3 same register
G & K	T6	4-9: 3,4,9,10	contiguous in both
I & J	IT3	pcs 5,10	contiguous in I
I & K	T4	pc10	
J & K	IT7	4-9: 3,4,9,10	contiguous in both (identical)

EXAMPLE

8 Invariant Subsets of Forms of 5-6

Sets	Transformation	Invariants	Comment
A & B	IT5	4-7: 0,1,4,5	contiguous (identical)
A & H	IT6	pcs 1,5	contiguous in H (first two notes)
A & L	T2	pc1	same register
B & H	T1	3-4; 1,5,6	contiguous in H
B & L	IT7	pcs 1,6	not contiguous
H & L	IT8	4-8: 1,2,6,7	not contiguous

Clearly, Example 7 contains too much information to lend itself to extensive treatment here. Three observations will suffice. First, there is no simple pattern for the transformations. Evidently there is no "systematic" application of a particular operation such as one might find in a 12-tone work by Webern. The only exception involves the three occurrences of the same form of pc set 4-9 as an invariant subset over three sets, G, J, and K. Its occurrence within G signals the end of the first main section of the external form. Finally, with one exception, the sum of pcs that results from combining any form of 5-7 with any other form of 5-7 is again that set. For instance, C is 10,11,0,4,5, and the sum of all the invariants that result from combining C with any other form of 5-7 is again C: 10, 11,0,4,5. This means that any pc within a form of 5-7 is held invariant over at least one other form of 5-7, and in this way the pc content of each form of 5-7 is preserved over the span of the work. The sole exception is E, the sum of the invariants of which is 4-9: 8,9,2,3.

Example 8 summarizes the relations among pairs of sets of type 5-6. Here further observation can be made concerning the distinction between 5-7 and 5-6. The sum of pcs for all forms of 5-7 excludes only one pitch-class, pc1, as noted above. This is exactly the pc that is invariant over all forms of 5-6, occurring four times: trombone, m.1; tuba, m.5; celesta, m.9; trombone, m.13. Pc set 5-6, however, excludes five pcs represented within forms of 5-7: 3,8,9,10, and 11. Thus, in this respect the two 5-element sets differ considerably with regard to pc content over the course of the entire piece. Indeed, apart from the local intersection in mm.1-3, the only invariant subset type common to both is 4-8, and there is no pc intersection involved in this correspondence.

Order relations for the 5-note sets are shown in Example 9. As in the case of the relations among the hexachords, the bips exhibit considerable similarity. All contain at least one 1, and six of the 14 bips have the subpattern 112. Only two bips contain an instance of interval-class 6. The same pcs are involved in both cases: B♭-E in m.2 and again in m.8, at the first large formal juncture.

Because they are repeated, three bips are especially significant: 1124, 1125, and 1115. Bip 1124 is formed by A and L, the first and last statements of pc set 5-6. Not only is the bip the same in each case, but also the interval succession is the same. It is not difficult to ascertain that L is an ordered transposition of A (t = 2). Bip 1124 is again formed by 1, the

EXAMPLE

9

Order Relations for 5-element Sets

	Interval Succession	Bip
5-6		
A: 11 1 0 4 5	2-1-4-1	1124
B: 1 0 4 5 6	1-4-1-1	1114
H: 1 5 6 2 7	4-1-4-5	1445
L: 1 3 2 6 7	2-1-4-1	1124
5-7		
C: 0 11 10 4 5	1-1-6-1	1116
D: 0 11 4 5 6	1-5-1-1	1115
E: 7 9 8 3 2	2-1-5-1	1125
F: 9 8 3 2 10	1-5-1-4	1145
D': 0 11 4 6 5	1-5-2-1	1125
G: 9 8 10 4 3	1-2-6-1	1126
I: 5 6 10 0 11	1-4-2-1	1124
J: 9 10 3 4 5	1-5-1-1	1115
K: 9 10 3 4 2	1-5-1-2	1125

statement of 5-7 that closes the next to last section of the piece, and the two successive and overlapping ordered 4-note sets in 1 are 4-8 (5 6 10 0) and 4-5 (6 10 0 11), the only subsets of size 4 that 5-7 shares with 5-6. (The ordering of 5-6, whether in A or L, does not bring out the corresponding 4-note sets, so that this aspect of the ordered relation among the three forms is less structured than is abstractly possible.)

Bip 1125 is formed by three statements of 5-7, E, D', and K. Whereas the interval succession of D' is almost the reverse of the interval succession of E, the interval succession of K is an exact reversal of the interval succession of E, and, indeed K is a retrograde of the ordered transposition of E:

E:	7 9 8 3 2	
		t=7
	2 4 3 10 9	(the ordered transposition of E)
K:	9 10 3 4 2	

To be sure, the ordered relation is probably extremely difficult to perceive since K is not only remote from E in the music but is also deployed in a way that obscures its identity as a structural unit. Nevertheless, order relations of this kind are of interest, and may be regarded as adumbrations of primitive 12-tone procedures.

The two occurrences of bip 1115 involve sets D and J. Here again, the interval succession is identical in both cases. Since the two sets are inversionally equivalent, one might expect that the second is an ordered inversion of the first. If that were true, however, J would be the pc succession 9 10 5 4 3 instead of 9 10 3 4 5.

We turn now to the 7-note complements of 5-6 and 5-7 displayed in Example 5 and consider two aspects of structure that these sets contribute to the whole: invariant subsets and embedded complements. For the sets of type 7-7, maximum invariance occurs between three pairs. (See Example 6.) A and B intersect in the hexachord 6-7, which, in terms of this analysis is not regarded as significant, since 6-7 is not a structural pc set in the music. A (or A') and E intersect in the structural hexachord 6-Z6: 11,0,1,4,5,6 while B and E intersect in 6-Z38: 4,5,6,7,11,0. Both these hexachords are shown in Example 1. Invariant subsets formed by the other pairs are all small. For A and D and for B and D the transposition level produces minimal invariance, and for the inversionally related pair D and E a single pc is held invariant (pc1). The latter was discussed in connection with the hexachords.

Of greater structural significance than invariants are the embedded complements of 7-7. All these are indicated in Example 5. For instance, form A of 7-7 contains two occurrences of its complement, 5-7, namely, C and D. In fact, each form of 7-7 contains at least one form of its complement.

In terms of number of occurrences, it would appear that the structural role of 7-6 is less important than that of 7-7. There is one occurrence in the first part (C) and one in the last part of the music (F). The two forms are transpositionally equivalent. Invariance is minimal and the invariant subset is of no consequence. Moreover, the embedded complement situation does not arise as in the case of 7-7. In the second part, however, 7-6 (F) is preceded immediately by its complement, 5-6 (H) and interlocks with its complement again at the end of the music. From Example 5 it is evident that 7-7 is the predominant 7-note set in the first part, while 7-6 is, in fact, the only structural 7-note set in the second part.

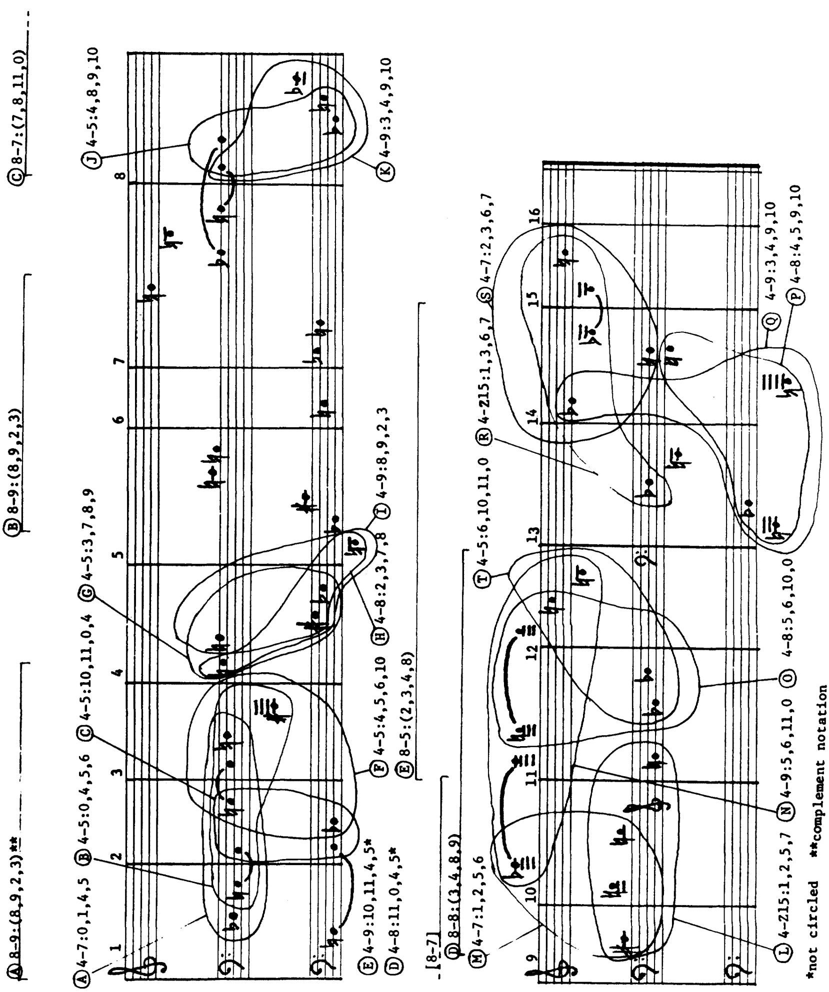
The Four- and Eight-Note Sets

Example 10 displays the 4- and 8-element sets. With the exception of 4-Z15, the complement of each 4-element set is also represented in the music, and with the exception of 4-Z15, every 4-element set is embedded at least once in some form of its complement. (The 4-element set 4-6: 10,11,0,5, which is invariant between hexachords 6-Z6 and 6-Z38 [F and G in Examples 1 and 2] is not included among the 4-element sets discussed here since it is not represented by its complement and is not prominent elsewhere in the music.)

Of the large 8-element sets, only 8-9 occurs more than once. Specifically, the first eight notes comprise 8-9 (A), and the eight notes beginning with the tuba B♭ in m.5 comprise the other form (B). It will be recalled that the D in m.5 is the end of a 12-note aggregate. This corresponds exactly to the juncture under discussion. The pc content of both forms is the same — that is, they are transpositionally equivalent, with t=0. These two identical forms are joined together by a form of 4-9 (1), or, to say this in another way, the opening twelve-note aggregate is partitioned into 8-9 and its literal complement, 4-9, and the latter is followed by a repetition of 8-9. Example 11, a display of the overall attack succession, shows the relations and provides a reference for the discussion that follows.

At m.9 there is a distinct break in continuity, both with respect

EXAMPLE 10



(A) 8-9:(8,9,2,3)**
(B) 8-9:(8,9,2,3)
(C) 8-7:(7,8,11,0)
(A) 4-7:0,1,4,5
(B) 4-5:0,4,5,6
(C) 4-5:10,11,0,4
(G) 4-5:3,7,8,9
(J) 4-5:4,8,9,10
(K) 4-9:3,4,9,10
(I) 4-9:8,9,2,3
(H) 4-8:2,3,7,8
(F) 4-5:4,5,6,10
(E) 8-5:(2,3,4,8)
(E) 4-9:10,11,4,5*
(D) 4-8:11,0,4,5*
[8-7]
(D) 8-8:(3,4,8,9)
(M) 4-7:1,2,5,6
(T) 4-5:6,10,11,0
(R) 4-Z15:1,3,6,7
(S) 4-7:2,3,6,7
(Q) 4-9:3,4,9,10
(P) 4-8:4,5,9,10
(O) 4-8:5,6,10,0
(N) 4-9:5,6,11,0
(L) 4-Z15:1,2,5,7
*not circled **complement notation

EXAMPLE 11

Webern, ORCHESTRAL PIECE 1913: Attack Succession

4-1 4-1 4-1 4-1

11 1 0 10 4 5 6 7 9 8 3 2 10 1 0 11 4 6 5 7

8-9 4-9 8-9

4-9 8-8

8 11 9 10 4 3 1 5 6 2 7 5 6 10 0 11 9 10 1 0

4-7 4-5

8-7 6-Z6

8-5

3 4 2 6 7
5

to external as well as internal form. The only structural set that links the two large divisions of the piece is 8-7 (9,10,1,2, 3,4,5,6), the complement of the first tetrachord (A). This 8-element set begins with pc9, the same note that ends the second 12-note aggregate mentioned above.

As shown in Example 10, this eight-note set (C) decomposes into two disjunct tetrachords: 4-9 (9,10,3,4) and 4-7 (1,2,5,6), labelled K and M, respectively. Beginning at m.9 a form of 8-8 (D) overlaps 8-7, incorporating as its first tetrachord 4-7 (1,2,5,6), and spanning the entire section, mm.9-12. The last tetrachord within 8-8 (D) is 4-5 (6,10,11,0), a subset of 6-Z6 (see Example 1, set F). Finally, as shown in Example 11, the eight-note set 8-5 interlocks with the last six notes of 8-8 (6-Z6), extending up to the entrance of the tremolo E♭ played by contrabass at m.14.

In the final section (mm.13-16), all previously stated tetrachords are represented, with the exception of 4-5. Of special interest is the final tetrachord, 4-7 (S). This is an ordered transposition of the first occurrence of 4-7 (A); consequently the two forms have the same interval succession (and bip). However, the transposition level is one of three such that produce complete variance with respect to pitch-class, and thus the correspondence does not extend to shared pcs.

Order relations and invariants among the tetrachords can be read from Example 12. At this level of structuring, of course, the relations are multiple and complicated – so much so that extended verbal commentary seems out of the question. A few observations concerning either particularly prominent or less obvious features will suffice.

Perhaps the most obvious feature of order relations shown in Example 12 is the marked similarity of bips (and, to a large extent, interval successions) not only within sets of the same type, but also ranging over the entire set of tetrachords. Many of the bips contain two 1's and all contain at least one 1, with the single exception of L (4-Z15). Indeed, the 29 tetrachords listed yield only 8 distinctive bips. Thus there is considerable uniformity (and potential ambiguity as well) among the bips. An extreme instance is found in the case of 4-9, which has bips of types 115 and 116 only. (4-9 is capable of producing only 6 distinct bips, in any event.) This high degree of bip correspondence might lead one to expect more specific order procedures for bips of the same type. This, however, is not the case. All the forms of 4-9 are transpositionally equivalent (in general,

EXAMPLE

12

Basic Interval Patterns for Tetrachords

	Succession	Bip
4-5		
B: 0 4 5 6	4-1-1	114
C: 11 0 10 4	1-2-6	126
F: 10 4 5 6	6-1-1	116
G: 7 9 8 3	2-1-5	125
J: 8 9 10 4	1-1-6	116
T: 6 10 0 11	4-2-1	124
4-7		
A: 1 0 4 5	1-4-1	114
M: 1 5 6 2	4-1-4	144
S: 3 2 6 7	1-4-1	114
4-8		
D: 11 0 4 5	1-4-1	114
H: 7 8 3 2	1-5-1	115
O: 5 6 10 0	1-4-2	124
P: 9 10 4 5	1-6-1	116
4-9		
E: 11 10 4 5	1-6-1	116
I: 9 8 3 2	1-5-1	115
K: 9 10 4 3	1-6-1	116
N: 6 5 0 11	1-5-1	115
Q: 9 10 3 4	1-5-1	115
4-Z15		
L: 1 5 2 7	4-3-5	345
R: 1 3 6 7	2-3-1	123

any inversion of a form of 4-9 is the same as some transposition of that form), but none of these is an ordered transposition. In fact, there is only one instance of an ordered transposition, and that involves forms S and A of 4-7, mentioned above. There is, however, one instance of retrograde inversion: among the forms of 4-5, J is the retrograde of the inversion of F transposed 2.

The many invariants among forms of sets of the same type can easily be ascertained from Example 10 or Example 12. For example, the prominent invariant dyads among forms of 4-5 are (4,10) and (8,9) and those dyads join at the end of the first large section to form the tetrachord 4-5 (J). As another instance, the dyad (4,5) occurs in the first statement of 4-8 (D) and again, with octave displacement, in the last statement of that set (P).

One peculiar result of this analysis is that mm.5-7 are entirely bereft of structural tetrachords. Structural relations for that part of the music occur within the context of sets 5, 6, and 8 notes. (See Example 1 and Example 5)

The chromatic set 4-1 is not included in Example 12. However, in Example 11 this tetrachord is shown to partition the two successive forms of 8-9, and, indeed, the pitch content of the tetrachords remains intact over both forms of 8-9. This tetrachord has not, however, been regarded as a unit of more fundamental significance since it is the result of the attack succession alone and is not represented in the more complicated texture that results from overlapping as well as contiguous notes. Hence, 4-1, although included in the set-complex analysis below, is a set that belongs to a single dimension.

In this connection, it should be pointed out that trichords are not emphasized in the present analysis, not because they are "unimportant," but merely because they are so obvious and because their structural roles are subsumed by those of the larger sets of which they are part. A trivial instance is the trichord 3-5, which occurs at the end of the section in m.9 (F-C-B). In both cases this trichord occurs within 4-9 (and larger sets), and, of course, all the 3-note subsets of 4-9 are reducible to trichords of type 3-5. More will be said about trichords below in connection with similarity relations.

Similarly, little attention has been given to the recurrence of specific pitches (or pcs), except insofar as these recurrences are meaningful with respect to the pc set components of the music as, for example, invariants (or "non-invariants"). This procedure leaves certain pitch-class functions unexplained. Perhaps the most evident example is provided by the last two notes of the composition: G♭-G. These notes occur within 4-7 (S), and it has already been pointed out that S in an ordered transposition of A, the first statement of 4-7 that begins in m.1. As a result of this relation the dyad G♭-G in S corresponds to the dyad E-F in A. This association of the close of the work with the opening is further strengthened by the only other occurrence of the enharmonically equivalent pitch succession, the dyad F♯-G, mm.3-4. In this context, F♯-G is immediately preceded by E-F.

The Abstract Set-Complex Relations

The preceding discussion has included a considerable amount of detail, representing an effort to show how this work is structured in various "dimensions" and over various spans. When the intricate nestings and intersections of pc sets are brought within the scope of the set complex, *8 a measure of simplification is introduced, however, for it then becomes possible to obtain an overview of the musical components, albeit one that is abstract with respect to the specific musical events in this work.

Example 13 provides a table from which one can read the set-complex relations of the pc sets represented in the work at hand. The fundamental structural role of the hexachordal pair 6-Z6/6-Z38 is evident immediately. All the other sets are in the set-complex relation K about this nexus pair, and four sets (4-5, 4-8, 4-9, and 5-7) are in the relation Kh to the hexachords 6-Z6/6-Z38.

The anomalous hexachord 6-Z44 is linked with the main set-complex through the five sets listed in its row in the table. Of these, 4-Z15, the first tetrachord in the statement of 6-Z44 (celesta, m.9), provides the immediate connection. Note also that 6-Z44 is within the large set 8-8 (D in Example 10).

In proceeding from the table of abstract relations back to the specific musical instances of those relations, it should be borne in mind that the inclusions need not be fully exemplified over every local context. For example, in one context, 7-7 (D in Example 5) contains 6-Z6 and 6-Z38 (D and C in Example 1, respectively), whereas 5-6 (L in Example 5) is not within a form of 6-Z6, but does contain 4-7 and 4-Z15 (S and R in Example 10, respectively).

Similarity Relations

Two sets of cardinal n that share some subset of cardinal n-1 are said to be in the similarity relation Rp. Rp is thus clearly associated with set-complex relations, and the existence of the relation for a particular pair of sets can be ascertained from the table of set-complex relations (Example 13). The two sets of cardinal 5, 5-6 and 5-7, share sets of types 4-5 and 4-8 (as stated above in connection with the discussion of Example 9). Thus similarity is a structural feature of the opening of the composition, where 5-6 and 5-7 (B and D in Example 5) share 4-5 (0,4,5,6) and 5-6 and 5-7 (A and C in Example 5) share 4-8 (11,0,4,5).

Similarity relations of type Rp elucidate to a certain extent the role of trichords in the work. Example 14 summarizes the common trichords of pairs of tetrachords. Trichords of types 3-4 and 3-5 predominate, with only a single instance of 3-3 at the very end of the piece. (In the case of the pairs 4-5, 4-8 and 4-8, 4-9, the relation R2 also holds – that is, the sets are maximally similar with respect to interval content as well.) In the opening section of the music these trichords fit neatly into the set-complex structure of the piece. As shown in Example 15, the common trichords exhaust two successive forms of 5-7 (cf. Example 5).

Orchestration

Exactly what position orchestration occupied in the order of Webern's composing process is not especially clear from the published sketches (for 12-tone works). *9 It is reasonable to assume, however, that orchestration was not a primary consideration, whereas the overall pitch organization of the music was a fundamental concern.

It also appears that orchestration, as well as other aspects of the music, such as dynamics and mode of performance (to be discussed briefly below) are far less "systematic" than is the pitch organization, hence less amenable to concise discussion. Accordingly, and in order to curtail unreasonable expansion of this already lengthy essay, only a few salient features of the orchestration will be mentioned here.

Perhaps the basic question is: to what extent is choice of instruments associated with the components of the structural pc sets? The answer in this case is: very little. The same is probably true for all of Webern's atonal works, as distinct

EXAMPLE

13

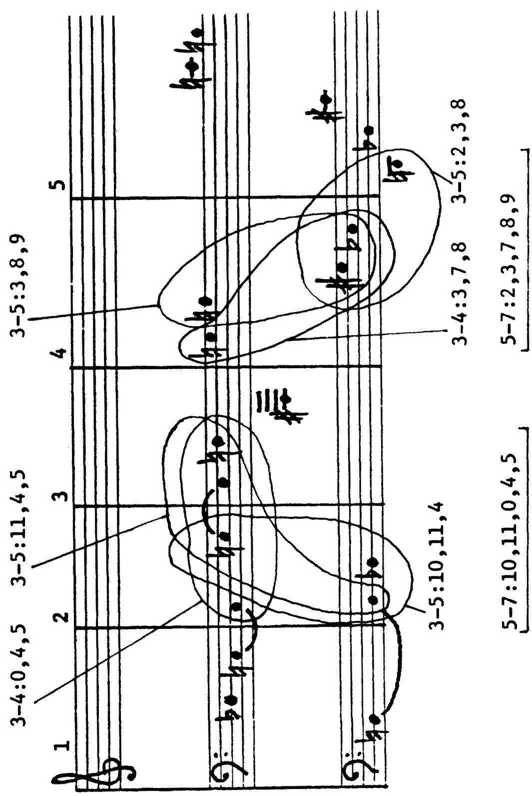
Set-complex Relations

	4-1	4-5	4-7	4-8	4-9	4-Z15	5-6	5-7
5-6	K	Kh	Kh	Kh	K	Kh		
5-7	K	Kh	K	Kh	Kh	K		
6-Z6/38	K	Kh	K	Kh	Kh	K	K	Kh
6-Z44		K	Kh	Kh		K	K	

14

Sets	Labels (Ex. 10)	Common Trichords	Trichord Type
4-5 & 4-8	B & D	0,4,5	3-4
	G & H	3,7,8	3-4
4-5 & 4-9	C & E	10,11,4	3-5
	G & I	3,8,9	3-5
4-7 & 4-8	A & D	0,4,5	3-4
4-7 & 4-Z15	R & S	3,6,7	3-3
4-8 & 4-9	D & E	11,4,5	3-5
	H & I	2,3,8	3-5

EXAMPLE 15


1
2
3
4
5
3-4:0,4,5
3-5:11,4,5
3-5:10,11,4
3-5:3,8,9
3-4:3,7,8
3-5:2,3,8
5-7:10,11,0,4,5
5-7:2,3,7,8,9

from the 12-tone works, where orchestration becomes more "structural." Whereas the underlying pitch organization of the music is relatively straightforward, in the sense that very few sets are employed, the orchestration presents a highly diversified sonic surface. Indeed, there are only two cases in which a pc is performed twice by the same instrument (pc1 [trombone] and pc4 [c. bassoon]), and in both cases the pc is rendered by other instruments as well. In short, there is no evidence of a systematic association of pc and instrument. Nor is the distribution of pcs over instruments uniform in any respect. Of the twenty instruments, ten are assigned one pc each. In contrast, the number of occurrences of each pc class is approximately the same. One pc occurs only twice (pc8), but the others range between three and five occurrences.

There is one feature of the orchestration that is perhaps evident from even brief inspection of the score. Each new section of what has been called the external form of the piece is introduced by a new timbre – that is, by a timbre that has not yet appeared. The succession is as follows:

bass clarinet	m. 1	(pc1)
English horn	m. 4	(pc7)
tuba	m. 5	(pc2)
horn	m. 7	(pc4)
celesta	m. 9	(pc1)
harmonium	m. 13	(pc9)

There is also a correspondence of instrument-type and the non-invariant pcs 1 and 7 that have been discussed above in connection with pc sets 6-Z6 and 6-Z38. This pattern is displayed below:

pc1 (m. 1)	pc7 (m. 4)	pc1 (m. 5)	pc7 (m. 7)
trombone	English horn	tuba	oboe

There is, however, no similar patterning in the remainder of the piece.

Rhythm

In the absence of a general model for rhythm that is appropriate for atonal music, it seems futile to make an extended contextual analysis here. Clearly, the intricate attack and duration patterns serve to differentiate the various pc set formations described above, but the precise way in which this is done eludes concise description – at least at the present time.

The complexity of the pattern formed by the successive attacks as well as the additional complexity introduced by diverse durations aggravates the task of supplying even a contextual description in the case of this particular piece. For instance, there are 18 durational units associated with distinct pcs, the longest being a dotted half note tied to whole note (b.cl., m.1, pc11), and the shortest, the single eighth note (harp, m.5, pc0, and elsewhere). There are only two cases in which a duration is assigned to the same pc more than once: the half note is assigned to pc2 twice and to pc5 twice. Furthermore, there is a wide range of difference in the assignment of durations to pcs. Of the 18 different durations, 10 are uniquely assigned, whereas in one case (the half note), the same duration is assigned to 9 different pcs.

Dynamics and Special Mode of Performance

By special mode of performance is meant any indicated way of performing other than that customary for the instrument – e.g., with mute, col legno, and so on. Again, as in the case of rhythm, an initial observation can be made, namely, that the differentiation provided by dynamics and mode of performance serves to emphasize the multiple dimensionality of the work. That is to say, these features (together with durations, of course) present the piece from being understood merely as a succession of attacks of equal value. As an illustration, let us consider the first section of the external form (mm.1-3). Example 1 shows that the hexachord 6-Z6 (A in Example 1) encompasses all the notes of this unit, with the exception of B♭. This note is set off from the others by mode of performance (mute and sul pont.) and belongs to the larger set, 7-7 (A in Example 5). As another instance, this time involving dynamics alone, consider 6-Z6 as it occurs in mm.11-12 (F in Example 1). The onset of this formation is signaled by the crescendo that begins on the first note, G. This crescendo and the subsequent complementary decrescendo bind together the three notes that form the trichordal subset (3-3) of 6-Z6. It is believed that further analysis of the piece along the lines of the two instances just cited will substantiate the following observation: dynamics and mode of performance are directly associated with the underlying pc set organization of the music. Once again, however, as in the case of rhythm, a useful language is lacking for the precise description of that association with reference to a general model of some kind.

REFERENCES

1 It is assumed that readers of this journal are familiar with most of the more common technical terms used in this article.

2 Segmentation procedures and problems are discussed in the author's THE STRUCTURE OF ATONAL MUSIC (Yale University Press, 1973) and in "Sets and Nonsets in Schoenberg's Atonal Music," PERSPECTIVES OF NEW MUSIC, 11 (1972).

3 A list of pc sets is given in THE STRUCTURE OF ATONAL MUSIC (see above).

4 The sets 6-Z6 and 6-Z38 are used by Webern in other atonal works – for example, in Op. 10/1, Op. 10/4, Op. 7/3. The 12-tone row of his first 12-tone composition, Op. 17, decomposes into the two hexachords 6-Z38 and 6-Z6.

5 As in the case of 6-Z6 and 6-Z38, 5-7 (and its complement) are used in other works of Webern – e.g., in Op. 9/5. This set is also favored by Schoenberg and is a structural set in certain works of Ives. It is unique among the 5-element sets in that its vector is maximally dissimilar from the vector of any other 5-element set; hence its total interval content is very distinctive.

6 The amount of invariance, of course, is a function of the value of the transposition operator (t).

7 See the author's "The Basic Interval Patterns," JOURNAL OF MUSIC THEORY, 17 (1973), 234-72.

8 See THE STRUCTURE OF ATONAL MUSIC (Reference 2).

9 Anton von Webern, SKETCHES (1926-1945), Carl Fischer, Inc., 1968.